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Robust Adaptive Control for Robotic Systems with Guaranteed Parameter Estimation

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Abstract. In most adaptive control systems with unknown parameters, the adaptive laws are usually designed based on the predictor/observer error or control error. In this paper, we propose a novel adaptive control scheme for nonlinear robotic systems by incorporating the parameter error into the adaptive law, which leads to guaranteed estimation performance. By carrying out filter operations, the robotic system can be linearly parameterized without using the measurements of acceleration. Then a new adaptive algorithm is introduced to guarantee that the parameter error and control error exponentially converge to zero. In particular, we provide an intuitive method to online verify the standard PE condition used for the parameter estimation. The robustness against disturbances is also studied and appropriate comparisons to several adaptive laws are provided. Simulations with a realistic robot arm are presented to validate the improved performance.

Keywords: Adaptive control, parameter estimation, robotic system, PE condition.

1. Introduction

Adaptive control [1] and robust adaptive control [2] have been widely usedto achieve tracking control for systems with unknown parameters. In the classical framework, the parameter adaptive laws are driven by the control errors to achieve the tracking error convergence and to remain the boundedness of the parameter estimation with e-modification and σ-modification [2]. However, it is not trivial to guarantee that the parameter estimations converge to their true values [3]. In particular, the required persistent excitation (PE) condition is generally difficult to verify. In [4, 5], a composite adaptive control has been developed to improve the control and estimation performance, where the adaptive laws are designed by combining the tracking error and the prediction error. However, since the adaptive laws are again driven by the induced output errors of controllers and predictions,

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the convergence performance heavily depends on the convergence of presented predictor and the overall control systems.

Recently, an alternative adaptive scheme has been proposed to address the parameter estimation problem, where the information of parameter error (the error between the unknown parameters and its estimated values) is explicated obtained and used to drive the adaptive laws [6]. In particular, in our previous work [7-9], several novel *direct* estimation schemes are proposed by introducing novel filter operations, so that the unstable integrator and the online calculation of a matrix inverse in [6] are all avoided. Moreover, exponential and/or finite-time error convergence are retained under a verify excitation condition.

With the development of modern industry, the demand for robotic manipulators has increased, and thus adaptive control of robotic systems has also been rapidly developed (e.g., [4, 10, 11]) since 1980s when the linear parameterization of nonlinear robot dynamics was introduced [4, 12]. In adaptive control proposed in [13], the inversion of the estimated inertia should be computed . In [11], the need of robotic joint acceleration measurements limits the practical applicability of such a control system [14]. Moreover, in these methods only the convergence of the tracking error can be proved, while the parameter estimation convergence has not been addressed. In our previous work [15], a terminal sliding mode (TSM) control was proposed for nonlinear robotic systems to achieve finite-time convergence by incorporating the ideas of adaptive laws [8] into the adaptive control, where the potential singularity problem ins TSM is avoided and the joint accelerations are not used by introducing new filer operations as [14]. However, this two-phase control leads to complexities in the analysis and practical implementations.

In this paper, we further revisit the adaptive control for robotic systems with unknown parameters, where the parameter estimation is also studied. The new adaptive control strategy will incorporate the parameter estimation methods proposed in [8] into the adaptive control to achieve exponential convergence of the tracking control and parameter estimation simultaneously. In particular, a new modification term with the estimation error is adopted as a new leakage term in the adaptive law. In contrary to [15], only one phase control is used to simplify the control implementation. The robustness of the proposed schemes against external disturbances is analyzed and comparisons to classical adaptive laws are provided. Specifically, we have proved that the required excitation condition is equivalent to the standard PE condition, and then suggested an intuitive scheme is to verify the standard PE condition. Finally, simulations based on a 6-DOF robot arm model are presented to validate the performance of the new method.

The paper is organized as follows: We start with studying the modeling of nonlinear robotic systems in Section 2; Section 3 introduces an adaptive control design for robotic systems. The robustness analysis and comparisons to other adaptations are presented in Section 4. Simulation results are provided in Section 5 and some conclusions are outlined in Section 6.

2. Problem formulation

An n-degree of freedom (DOF) robot arm is a highly nonlinear system. In this paper, we consider a general structure of the robot systems modeled by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are the robot arm joint position, velocity and acceleration, respectively; n is the number of the DOF of robot, $\tau \in \mathbb{R}^n$ is the control input torque; $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents the Coriolis/ centripetal torque, viscous and nonlinear damping, and $G(q) \in \mathbb{R}^n$ denotes the gravity torque.

The following properties will be used in the adaptive control design [12]:

Property 1. *The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, then*

$$x^T \left[\dot{M}(q) - 2C(q,\dot{q})\right] x = 0, \qquad x, q, \dot{q} \in \mathbb{R}^n \tag{2}$$

Property 2. *The dynamics of robotic system (1) can be represented as a linearly parameterized form*

$$M(q)\ddot{q} + C(q,\dot{q})q + G(q) = \phi(q,\dot{q},\ddot{q})\theta \tag{3}$$

where $\theta \in \mathbb{R}^N$ is a constant parameter vector which contains the parameters to be estimated, $\phi(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times N}$ is the known dynamic regressor matrix.

Definition 1. [1] *A vector or matrix function ϕ is persistently excited (PE) if there exist $T > 0, \varepsilon > 0$ such that $\int_{\tau}^{\tau+T} \phi(r)\phi^T(r)dr \geq \varepsilon I, \ \ \forall \tau \geq 0$*

3. Adaptive control with guaranteed parameter estimation

For the sake of achieving the tracking control for a given reference q_d and the estimation of unknown parameter θ, the parameter error will be obtained and then used in the control design of robotic system (1) in this section.

We first define a control error variable as

$$S = \dot{e} + \lambda e = \dot{q}_r - \dot{q} \tag{4}$$

where $e = q_d - q$; $\dot{q}_r = \dot{q}_d + \lambda e$ is the tracking error and its derivative $\ddot{q}_r = \ddot{q}_d + \lambda \dot{e}$ can be calculated based on $q, \dot{q}, \dot{q}_d, \ddot{q}_d$.

Then based on Property 2, we define

$$R(q,\dot{q}) = M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) = \Phi_R(q,\dot{q})\theta \tag{5}$$

Then the robotic system (1) can be reformulated as

$$M(q)\dot{S} + C(q,\dot{q}) - R(q,\dot{q}) = -\tau \tag{6}$$

As shown in (5), the joint acceleration measurement $\ddot{q}$ does not appear in the regressor $\Phi_{\mathrm{R}}(q,\dot{q})$, thus $\Phi_{\mathrm{R}}(q,\dot{q})$ can be used in the control design. However, for the purpose of parameter estimation with (6), the joint acceleration $\ddot{q}$ is involved in the derivative $\dot{S}$. To eliminate the requirements of $\dot{S}$, we define auxiliary functions $F(q,\dot{q}) = M(q)S$ and $H(q,\dot{q}) = -\dot{M}(q)S + C(q,\dot{q})S$ as [14] as

$$\begin{cases} F(q,\dot{q}) = M(q)S = \Phi_F(q,\dot{q})\theta \\ H(q,\dot{q}) = -\dot{M}(q)S + C(q,\dot{q})S = \Phi_H(q,\dot{q})\theta \end{cases} \tag{7}$$

Then, the system (6) can be rewritten as

$$\dot{F}(q,\dot{q}) + H(q,\dot{q}) - R(q,\dot{q}) = [\dot{\Phi}_F(q,\dot{q}) + \Phi_H(q,\dot{q}) - \Phi_R(q,\dot{q})]\theta = \Phi(q,\dot{q},\ddot{q})\theta = -\tau \tag{8}$$

where $\dot{F}(q,\dot{q}) = \dfrac{d}{dt}[M(q)S] = \dot{\Phi}_F(q,\dot{q})\theta$ is the derivative of $M(q)S$ and $\Phi(q,\dot{q},\ddot{q}) = [\dot{\Phi}_F(q,\dot{q}) + \Phi_H(q,\dot{q}) - \Phi_R(q,\dot{q})]$ is the regressor matrix. However, because $\dot{F}(q,\dot{q})$ and thus the regeressor $\Phi(q,\dot{q},\ddot{q})$ contain the joint acceleration $\ddot{q}$, Eq.(8) cannot be used for parameter estimation when the joint acceleration $\ddot{q}$ is not measurable.

To eliminate the use of $\ddot{q}$ in the parameter estimation, we introduce a stable filter operation on both sides of (8) as

$$\begin{cases} k\dot{\Phi}_{Ff} + \Phi_{Ff} = \Phi_F, & \Theta_{Ff}\,|_{t=0} = 0 \\ k\dot{\Phi}_{Hf} + \Phi_{Hf} = \Phi_H, & \Theta_{Hf}\,|_{t=0} = 0 \\ k\dot{\Phi}_{Rf} + \Phi_{Rf} = \Phi_R, & \Theta_{Rf}\,|_{t=0} = 0 \\ k\dot{\tau}_f + \tau_f = \tau, & \tau_f\,|_{t=0} = 0 \end{cases} \tag{9}$$

where $k > 0$ is a constant filter parameter, $\Phi_{Ff}(q,\dot{q}), \Phi_{Hf}(q,\dot{q}), \Phi_{Rf}(q,\dot{q})$ and $F_f(q,\dot{q}), H_f(q,\dot{q}), R_f(q,\dot{q}), \tau_f$ are the filtered form of $\Phi_F(q,\dot{q}), \Phi_H(q,\dot{q}), \Phi_R(q,\dot{q})$ and $F(q,\dot{q}), H(q,\dot{q}), R(q,\dot{q}), \tau$, respectively. Then from (8)~(9), we obtain

$$\dot{F}_f(q,\dot{q}) + H_f(q,\dot{q}) - R_f(q,\dot{q}) = [\dot{\Phi}_{Ff}(q,\dot{q}) + \Phi_{Hf}(q,\dot{q}) - \Phi_{Rf}(q,\dot{q})]\theta = -\tau_f \tag{10}$$

According to the first equation of (9), we get $\dot{\Phi}_{Ff} = \dfrac{\Phi_F - \Phi_{Ff}}{k}$, sot that the system (10) can be rewritten as

$$\left[\frac{\Phi_F(q,\dot{q}) - \Phi_{Ff}(q,\dot{q})}{k} + \Phi_{Hf}(q,\dot{q}) - \Phi_{Rf}(q,\dot{q})\right]\theta = \Phi_f(q,\dot{q})\theta = -\tau_f \tag{11}$$

where $\Phi_f(q,\dot{q}) = \dfrac{\Phi_F(q,\dot{q}) - \Phi_{Ff}(q,\dot{q})}{k} + \Phi_{Hf}(q,\dot{q}) - \Phi_{Rf}(q,\dot{q})$ is a new regressor. In this case, the acceleration $\ddot{q}$ is avoided and (11) will be used for estimation.

In the following developments, we will accommodate the parameter estimation by introducing the auxiliary matrix $P(t) \in \mathbb{R}^{N\times N}$, vector $Q(t) \in \mathbb{R}^N$ and vector

$W(t) \in \mathbb{R}^N$ as

$$\begin{cases} \dot{P}(t) = -\ell P(t) + \Phi_f^T \Phi_f, & P(0) = 0 \\ Q(t) = -\ell Q(t) + \Phi_f^T \tau_f, & Q(0) = 0 \\ W(t) = P(t)\hat{\theta} - Q(t) \end{cases} \tag{12}$$

where $\ell > 0$ is a designed parameter, $\hat{\theta}$ is the estimated parameter.

As shown in [9], we know $Q(t) = P(t)\theta$, and thus derive from (11)~(12) that

$$W(t) = P(t)\hat{\theta} - Q(t) = P(t)\hat{\theta} - P(t)\theta = -P(t)\tilde{\theta} \tag{13}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter estimation error.

Then we can design an adaptive control for system (1) as

$$\tau = \Phi_R(q, \dot{q})\hat{\theta} + KS \tag{14}$$

where K>0 is a constant feedback gain matrix.

The adaptive law to update the parameter estimation $\hat{\theta}$ is provided by

$$\dot{\hat{\theta}} = \Gamma\left(\Phi_R^T(q, \dot{q})S - \kappa W\right) \tag{15}$$

where $\Gamma > 0$ and $\kappa > 0$ are the adaptive learning gains.

Substituting (14) into (6), we obtain the closed-loop error dynamics as

$$M(q)\dot{S} + C(q, \dot{q})S + KS = \Phi_R(q, \dot{q})\tilde{\theta} \tag{16}$$

Remark 1: The term W in (15) is a new leakage term imposed in the adaptive law (15), which were used to achieve parameter estimation convergence (as will be shown later). This is clearly different to classical e-modification and σ-modification [2]. Moreover, by introducing filter operation (9), system (8) is reformulated as (11), so that the joint acceleration $\ddot{q}$ is avoided in both of the adaptive control (14) and the adaptive law (15).

To prove the error convergence of (15) and (16), we need to analyze the positive definiteness of matrix $P(t)$ in (12) as:

Lemma 1. *If the vector Φ_f in (8) is persistently excited (PE), then the matrix P in (12) is positive definite, i.e., $\lambda_{\min}(P) > \sigma > 0$ for a positive constant σ. On the other hand, the positive definiteness of P also implies that Φ_f is PE.*

Proof: From Definition 1, the PE condition $\int_t^{t+\tau} \Phi_f^T(r)\Phi_f(r)dr \geq \varepsilon I$ is equivalent to $\int_{t-\tau}^{t} \Phi_f^T(r)\Phi_f(r)dr \geq \varepsilon I$ for $t > \tau > 0$.

We first prove that if Φ_f is PE, then P is positive definite ($\lambda_{\min}(P) > \sigma > 0$). Based on the above analysis, if Φ_f is PE, then $\int_{t-\tau}^{t} \Phi_f^T(r)\Phi_f(r)dr \geq \varepsilon I$ holds for $t > \tau > 0$, such that the following inequality is true

$$\int_{t-\tau}^{t} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr \geq \int_{t-\tau}^{t} e^{-\ell\tau}\Phi_f^T(r)\Phi_f(r)dr \geq e^{-\ell\tau}\varepsilon I \quad (17)$$

which is obtained via the fact $e^{-\ell(t-r)} \geq e^{-\ell\tau} > 0$ for the integral interval $r \in [t-\tau, t]$.

Moreover, it can be verified for all $t > \tau > 0$ that

$$\int_{0}^{t} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr > \int_{t-\tau}^{t} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr \quad (18)$$

From (17)~(18), one can conclude that

$$P = \int_{0}^{t} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr > e^{-\ell\tau}\int_{t-\tau}^{t} \Phi_f^T(r)\Phi_f(r)dr \geq e^{-\ell\tau}\varepsilon I \quad (19)$$

This implies that P is positive definite and thus $\lambda_{\min}(P) > \sigma > 0$ for $\sigma = e^{-\ell\tau}\varepsilon$.

Now, we will prove that if P is positive definite, then Φ_f is PE. If P is positive definite, i.e., $P_1 = \int_{0}^{t} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr \geq \varepsilon I$ for positive constant ε, it follows

$$\begin{aligned}\varepsilon I &\leq \int_{0}^{t} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr \\ &= \int_{0}^{t-\tau} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr + \int_{t-\tau}^{t} e^{-\ell(t-r)}\Phi_f^T(r)\Phi_f(r)dr \\ &\leq \frac{e^{-\ell\tau}}{\ell}\|\phi_{1f}\|_\infty I + \int_{t-\tau}^{t} \Phi_f^T(r)\Phi_f(r)dr\end{aligned} \quad (20)$$

The last inequality is obtained from the use of $\int_{0}^{t-\tau} e^{-\ell(t-r)}dr \leq e^{-\ell\tau}/\ell$. Thus Eq.(4) gives

$$\int_{t-\tau}^{t} \Phi_f^T(r)\Phi_f(r)dr \geq \varepsilon^{\dagger} I, \quad \text{for } t \geq \tau \quad (21)$$

where $\varepsilon^{\dagger} = \varepsilon - e^{-\ell\tau}\|\phi_{1f}\|_\infty/\ell > 0$ for sufficiently large ℓ and τ. This implies that the regressor matrix Φ_f is PE.

Remark 2: In all of our previous work [7-9, 15], we have only proved that the standard PE condition is *sufficient* to guarantee positive definiteness of matrix $P(t)$, i.e., the PE condition of Φ_f implies $\lambda_{\min}(P) > \sigma > 0$. However, the inverse of this claim (i.e., $\lambda_{\min}(P) > \sigma > 0$ implies the PE of Φ_f) was not addressed. In this paper, the second claim has been formally proved, and thus Lemma 1 in this paper is a more complete result paving a way for online verifying the PE condition, i.e., calculating the minimum eigenvalue of P and testing for $\lambda_{\min}(P) > \sigma > 0$. This scheme is numerically feasible. To the best of our knowledge, the issue for online verification of PE condition has rarely been solved in the literature.

Now we have the following results:

Theorem 1. *For robotic system (1) with adaptive control (14) and adaptive law (15), if the regressor matrix Φ_f in (8) is PE, then the parameter error $\tilde{\theta}$ and the tracking error S converge to zero exponentially.*

Proof: We define Lyapunov candidate function as

$$V = \frac{1}{2}S^T MS + \frac{1}{2}\tilde{q}^T \Gamma^{-1}\tilde{q} \tag{22}$$

Then the derivative $\dot{V}$ with respect to time *t* can be obtained along (15)~(16) as

$$\begin{aligned}\dot{V} &= S^T\left[-C(q,\dot{q})S - KS + \Phi_R\tilde{\theta}\right] + \frac{1}{2}S^T\dot{M}(q)S + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \\ &= -S^T KS - \kappa\tilde{\theta}^T P\tilde{\theta} \\ &\leq -\mu V\end{aligned} \tag{23}$$

where $\mu = \min\left\{2\lambda_{\min}(K)/\lambda_{\max}(M), 2\kappa\sigma/\lambda_{\max}(\Gamma^{-1})\right\}$ is a positive constant. Then according to Lyapunov theory, we conclude that S and $\tilde{\theta}$ are bounded and converge to zero exponentially with the convergence rate μ.

Remark 3: As shown in Theorem 1, by introducing the leakage term κW containing the parameter estimation error $P(t)\tilde{\theta}$ in the adaptive law (15), a quadratic term $\kappa\tilde{\theta}^T P\tilde{\theta}$ appears in the Lyapunov analysis (23), which can guarantee the exponential convergence of $\tilde{\theta}$ and S to zero simultaneously.

4. Robustness analysis and comparisons

We will study the s robustness of the proposed control and adaptive law. For this purpose, we introduce a bounded disturbance $\xi \in \mathbb{R}^n$ in the robotic system (1) such that it can be presented as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \xi \tag{24}$$

where $\|\xi\| \leq \varepsilon_\xi, \varepsilon_\xi > 0$.

Then similar to Section 3, system (24) can be rewritten as

$$\Phi_f(q,\dot{q})\theta = \tau_f + \xi_f \tag{25}$$

where ξ_f is the filtered version of ξ in terms by $k\dot{\xi}_f + \xi_f = \xi,\ \xi_f(0) = 0$.

Consequently, by defining the control error as (4) and using the same control (14), we can obtain the closed-loop error for (24) as

$$M(q)\dot{S} + C(q,\dot{q})S + KS = \Phi_R(q,\dot{q})\tilde{\theta} + \xi \tag{26}$$

On the other hand, the auxiliary variable *W* defined in (12) can be represented as

$$W = P\hat{\theta} - Q = -P\tilde{\theta} + \psi \tag{27}$$

where $\psi = -\int_0^t e^{-\ell(t-r)}\Phi_f^T(r)\xi_f(r)dr$ is bounded by $\|\psi\| \leq \varepsilon_\psi$ for some $\varepsilon_\psi > 0$.

Then we have the following results:

Corollary 1. *Consider system (24) with control (14) and adaptive law (15), if the regressor matrix* Φ_f *in (8) is PE, then the closed-loop system is stable, and the estimation error* $\tilde{\theta}$ *and the tracking error* S *converge to a small set around zero.*

Proof: Consider the Lyapunov function as

$$V_2 = \frac{1}{2}S^T MS + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} \tag{28}$$

We know $\lambda_{\min}\left(P(t)\right) > \sigma > 0$ holds, so that $\dot{V}_2$ can be calculated as

$$\begin{aligned}\dot{V}_2 &= S^T\left[-C(q,\dot{q})S - KS + \Phi(q,\dot{q})\tilde{\theta} + \xi\right] + \frac{1}{2}S^T\dot{M}S + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ &= -S^T KS + S^T\xi - \kappa\tilde{\theta}^T P\tilde{\theta} + \tilde{\theta}^T\psi \\ &= -\left(\lambda_{\min}(K) - \frac{1}{2\eta}\right)\|S\|^2 - (\kappa\sigma - \frac{1}{2\eta})\|\tilde{\theta}\|^2 + \frac{\eta\varepsilon_\xi^2}{2} + \frac{\eta\varepsilon_\psi^2}{2} \\ &\le -\mu_2 V_2 + \gamma_2\end{aligned} \tag{29}$$

where $\mu_2 = \min\left\{2(\lambda_{\min}(K) - 1/2\eta)/\lambda_{\max}(M), 2(\kappa\sigma - 1/2\eta)/\lambda_{\max}(\Gamma^{-1})\right\}$ and $\gamma_2 = \eta\varepsilon_\xi^2/2 + \eta\varepsilon_\psi^2/2$ are all positive constants for large $\eta > 0$. Based on the results in [16], we know that all signals in the closed-loop system are bounded, and the errors $\tilde{\theta}$ and S converges to small residual set around zero.

Finally, we will compare the proposed novel adaptive law (15), with widely used gradient method and σ-modification .

1) Gradient method [1]: The adaptive law for parameter estimation is solely driven by the tracking error S , i.e., the parameter $\kappa = 0$ in (15). Then we can obtain the estimation error as

$$\dot{\tilde{\theta}} = -\Gamma\Phi_R^T(q,\dot{q})S \tag{30}$$

A critical problem of the gradient adaptation is the lack of guaranteeing the convergence of the estimation error $\tilde{\theta}$. This can be intuitively explained based on (30), i.e., the convergence of $\tilde{\theta}$ to zero cannot be claimed even the tracking error S converges to zero. This may lead to bursting phenomenon.

2) σ *-modification [2]*: A modification term $\kappa\hat{\theta}$ is used to replace the term κW in (15) to give the σ -modification method

$$\dot{\hat{\theta}} = \Gamma\left(\Phi_R^T(q,\dot{q})S - \kappa\hat{\theta}\right) \tag{31}$$

Then the estimation error can be obtained as

$$\dot{\tilde{\theta}} = -\Gamma\Phi_R^T(q,\dot{q})S + \Gamma\kappa\hat{\theta} = -\Gamma\kappa\tilde{\theta} - \Gamma\Phi_R^T(q,\dot{q})S + \Gamma\kappa\theta \tag{32}$$

The error equation (32) contains a term $\Gamma\kappa\tilde{\theta}$ that is linear in $\tilde{\theta}$, thus one can claim that the error dynamics in (32) are bounded-input-bounded-output (BIBO), i.e., the estimation error $\tilde{\theta}$ is bounded for bounded S and θ . However, the involved damping term $\Gamma\kappa\theta$ in (31) makes the estimated parameter $\hat{\theta}$ stay in the neighborhood of the pre-selected value only, rather than converge to its real value. In fact, when the tracking error $S=0$, the transfer function of (32) from θ to $\tilde{\theta}$ is $\tilde{\theta}=\dfrac{\Gamma\kappa\theta}{p+\Gamma\kappa}$ (p is the Laplace operation). Consequently, the ultimate bound of $\tilde{\theta}$ cannot be null.

3) Proposed method: In this paper, a new leakage term κW is included in the adaptive law (15), then the estimation error of can be given as:

$$\dot{\tilde{\theta}}=-\Gamma\kappa P\tilde{\theta}-\Gamma\Phi_R^T(q,\dot{q})S \tag{33}$$

It is shown that the error equation (33) introduces a forgetting factor $\Gamma\kappa P\tilde{\theta}$ as σ-modification (32). Thus, the error $\tilde{\theta}$ is also BIBO stable. Consequently, the robustness of the proposed adaptive law is compatible to σ-modification (32).

Moreover, the leakage term κW can update the estimated parameter $\hat{\theta}$ towards its true value θ. In fact, the error equation (33) is represented as $\tilde{\theta}=\dfrac{\Gamma\Phi_R^T S}{p+\Gamma P\kappa}$, so that $\tilde{\theta}\to 0$ holds as long as $S\to 0$. Thus, the adaptive law (15) can obtain better estimation performance than σ-modification (32).

5. Simulation

In this paper, we use the derived model of a 6-DOF Robot Arm (which is purchased from Reinovo Ltd) for simulation. In order to simply the control design and to illustrate the parameter estimation performance, only two joints (joint 1 and joint 2 in Figure 1) of this robot arm are modeled in this paper.

According to [17], we deduce the kinetic energy $\mathcal{K}$ and the potential energy $\mathcal{P}$ as

$$\begin{aligned}\mathcal{K}&=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2\\&=\frac{1}{2}m_1l_1^2\dot{q}_1^2+\frac{1}{2}m_2l_2^2\dot{q}_1^2+\frac{1}{2}m_2l_2^2\dot{q}_2^2+m_2l_2^2\dot{q}_1\dot{q}_2\\\mathcal{P}&=mgh\\&=m_1g(l_1\sin q_1+320)+m_2g[l_2\sin(q_1+q_2)+l_1\sin(q_1)+320]\end{aligned} \tag{34}$$

where v and ω are the velocity and angular velocity respectively. I is the moment of inertia and h is the height.

The Lagrange's equation of a robotic system is given by:

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\Theta}}-\frac{\partial\mathcal{L}}{\partial\Theta}=\tau \tag{35}$$

where $\mathcal{L}=\mathcal{K}-\mathcal{P}$ is the Lagrangian.

Then the dynamics of the robotic arm is molded as

$$\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}+\begin{bmatrix} C_{11}(q,\dot{q}) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q,\dot{q}) \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}+\begin{bmatrix} G_1(q) \\ G_2(q) \end{bmatrix}=\tau \tag{36}$$

with

$M_{11}(q)=m_2l_2^2+(m_1+m_2)l_1^2+2m_2l_1l_2\cos(q_2), M_{12}(q)=M_{21}(q)=m_2l_2^2+m_2l_1l_2\cos(q_2)$

$M_{22}(q)=m_2l_2^2, C_{11}(q,\dot{q})=-2m_2l_1l_2\dot{q}_2\sin(q_2), C_{12}(q,\dot{q})=-m_2l_1l_2\dot{q}_2\sin(q_2)$

$C_{21}(q,\dot{q})=m_2l_1l_2\dot{q}_1\sin(q_2), C_{22}(q,\dot{q})=0, G_1(q)=m_2gl_2\cos(q_1+q_2)+(m_1+m_2)gl_1\cos(q_1)$

$G_2(q)=m_2gl_2\cos(q_1+q_2)$

where m_1,m_2 are the mass of robot arm, l_1,l_2 are the length of each link, $g=9.18$ is the gravity constant.

Figure 1. 6-DOF Robot Arm

In this study, we assume the unknown parameters to be estimated in system (36) is $\theta=[m_1,m_2]^T=[10,5]^T$, and the auxiliary variables $F(q,\dot{q}),H(q,\dot{q}),R(q,\dot{q})$ can be derived as

$$R(q,\dot{q})=M(q)\ddot{q}_r+C(q,\dot{q})\dot{q}_r+G(q)$$

$$=\underbrace{\begin{bmatrix} l_1^2\ddot{q}_{r1}+l_1g\cos(q_1) & \begin{matrix}(l_1^2+l_2^2+2l_1l_2\cos(q_2))\ddot{q}_{r1}+(l_1l_2\cos(q_2)+l_2^2)\ddot{q}_{r2}+l_1g\cos(q_1)\\ -2l_1l_2\sin(q_2)\dot{q}_2\dot{q}_{r1}-l_1l_2\sin(q_2)\dot{q}_2\dot{q}_{r2}+l_2g\cos(q_1+q_2)\end{matrix} \\ 0 & \begin{matrix}(l_1l_2\cos(q_2)+l_2^2)\ddot{q}_{r1}+l_2^2\ddot{q}_{r2}\\ +l_1l_2\sin(q_2)\dot{q}_1\dot{q}_{r1}+l_2g\cos(q_1+q_2)\end{matrix} \end{bmatrix}}_{\Phi_R(q,\dot{q},\dot{q}_r,\ddot{q}_r)}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \tag{37}$$

$$F(q,\dot{q})=\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \end{bmatrix}=\begin{bmatrix} M_{11}(q)S_1+M_{12}(q)S_2 \\ M_{21}(q)S_1+M_{22}(q)S_2 \end{bmatrix}$$
$$=\underbrace{\begin{bmatrix} l_1^2 S_1 & (l_1^2+l_2^2+2l_1l_2\cos(q_2))S_1+\frac{1}{12}(l_1l_2\cos(q_2)+l_2^2)S_2 \\ 0 & (l_1l_2\cos(q_2)+l_2^2)S_1+l_2^2S_2 \end{bmatrix}}_{\phi_F(q,\dot{q})}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \tag{38}$$

$$H(q,\dot{q})=-\dot{M}(q)S+C(q,\dot{q})S$$
$$=-\begin{bmatrix} \dot{M}_{11}(q)S_1+\dot{M}_{12}(q)S_2 \\ \dot{M}_{21}(q)S_1+\dot{M}_{22}(q)S_2 \end{bmatrix}+\begin{bmatrix} C_{11}(q,\dot{q})S_1 \\ C_{21}(q,\dot{q})S_1 \end{bmatrix} \tag{39}$$
$$=\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & l_1l_2S_1(\dot{q}_1+\dot{q}_2)\sin(q_2) \end{bmatrix}}_{\phi_H(q,\dot{q})}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

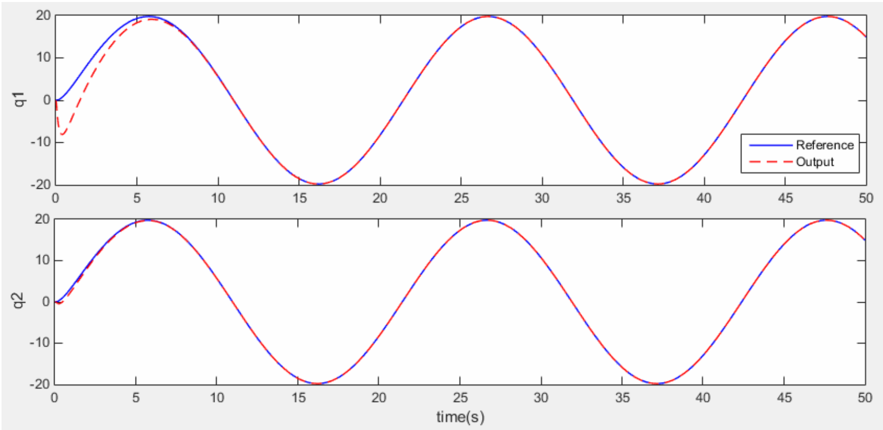
where $S=[S_1,S_2]^T$ is the control error with the parameter as $\lambda = diag([5,5])$.

To guarantee the required excitation condition, we choose the reference as $q_d = 20\sin(0.3t)$, and we set the control feedback gain as $K = diag([10,10])$. The parameters used in adaptive laws are set as $\ell = 1, k = 0.001, \kappa = 50$ and $\Gamma = 20I$. Comparative simulation results are shown in Figure 2, where the adaptive control (14) and the proposed adaptive law (15) and the gradient method (30) and σ-modification (31) are all simulated with the same condition and parameters.

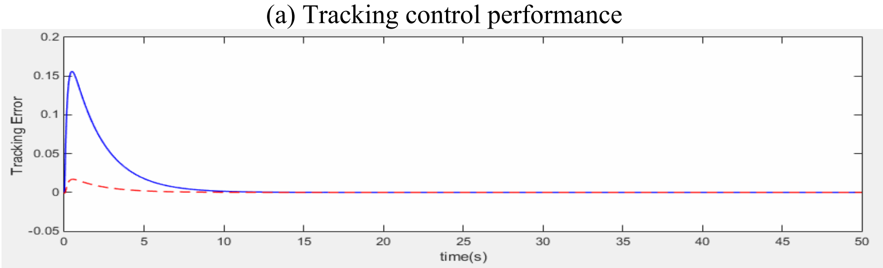
The output tracking profiles and the tracking error of the proposed methods (14) ~(15) are shown in Figure.2 (a)-(b), where fairly good control performance is achieved. The evolutions of the estimated parameters with different adaptive laws are all depicted in Figure 2 (c). It is clearly shown that the estimated parameters with new adaptive law (15) converge to their true values very fast as we claimed in Theorem 1. However, the transient convergence performance for the gradient scheme is sluggish and with significant oscillations. Nevertheless, the steady-state error for σ-modification method (31) cannot converge to zero although its transient convergence is similar to that of the gradient scheme, which has been pointed out in Section 4.

Finally, a bounded external disturbance $\xi = 0.2\sin(t)$ is applied on the control system to verify the robustness analysis as detailed in Section 4. As shown in Figure 2(d), the parameter errors of the new adaptive control methods converge to a very small set as well as the tracking error. However, the gradient method provides sluggish control and estimation performance though it performs slightly better than the σ-modification method.

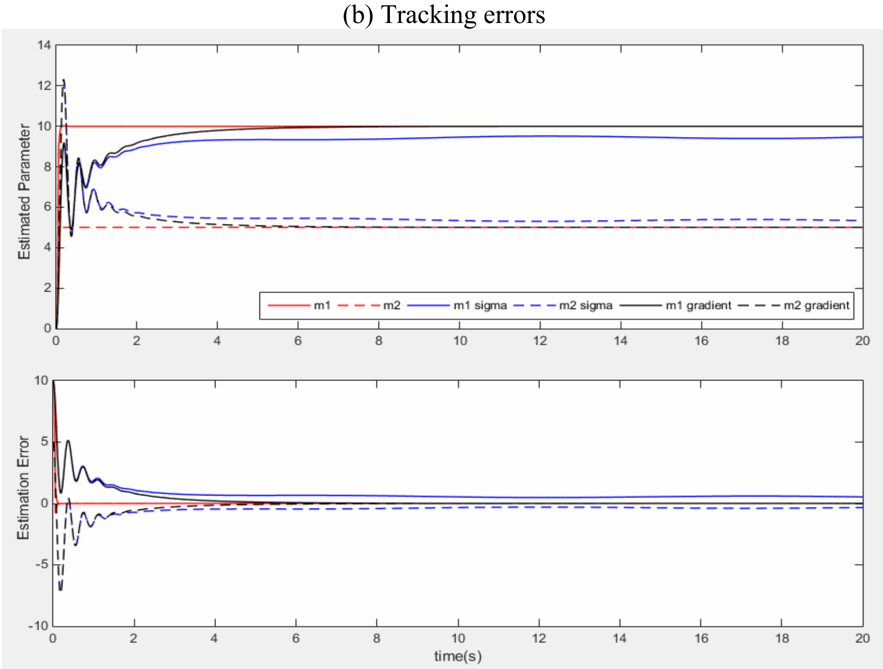
All of above simulation results show that the newly introduced leakage term in the adaptive law can improve the parameter estimation and thus the overall control performance. Moreover, the predictor used in the composite control scheme [3] is not used, and the joint accelerations are also avoided.



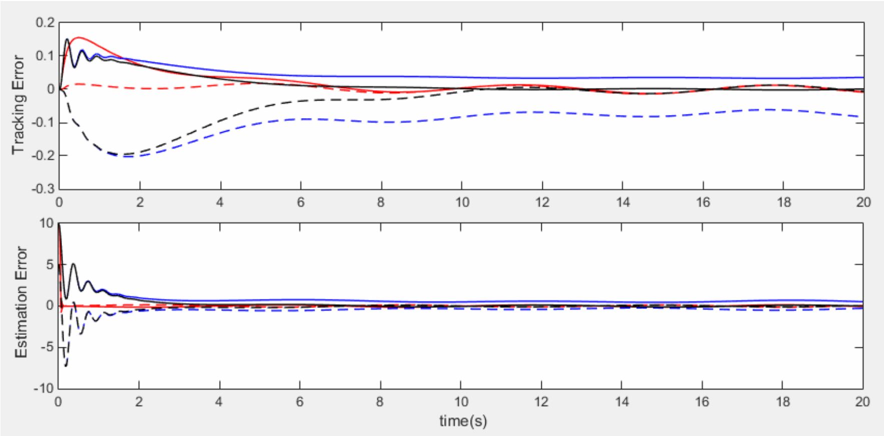
(a) Tracking control performance



(b) Tracking errors



(c) Parameter estimation performance



(d) Tracking and estimation errors under disturbances

Figure 2. Adaptive control and parameter estimation performance.

6. Conclusion

This paper presents an alternative adaptive control method for robotic systems, which incorporates a new leakage term into the adaptive law. By introducing appropriate filter operations, the robotic acceleration measurements are avoided. Exponential convergence of the control error and parameter estimation error to zero can be achieved simultaneously. In particular, we prove that the required excitation condition is equivalent to the standard PE condition, and thus provide a numerically feasible and intuitive method to online verify the PE condition. The robustness and comparisons to other adaptive schemes are also provided and validated in terms of simulations based on a realistic robotic arm model.

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